

# Mathematically Gifted in the Heterogeneously Grouped Mathematics Classroom:

## What is a Teacher to Do?

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Differentiation provides one method by which teachers can provide appropriate challenges at appropriate levels for all learners in a heterogeneously grouped mathematics classroom, where the range of abilities and interests can be wide. This article considers a heterogeneously grouped high school geometry class where differentiation is practiced. Students who demonstrated mastery of the concepts and skills still under study are invited to move into a differentiated option closely linked to the current class material. Three differentiation opportunities are presented and discussed. The first opportunity is an extension and application of current class work. The second is an investigation of open-ended questions. The third is a consideration of student-selected problems. Each provides content, process, and product differentiation.

It is the first day of the new school year. I face many challenges as the teacher in this first-period class, a heterogeneously organized geometry class. Now in my 17th year of teaching mathematics, I am confident that I have sufficient education and experience to offer my students a rich and appropriately rigorous course. Today there are the usual “start-up” administrative activities and normal classroom management issues, and there are also issues of how best to assess the students’ skills, how to adjust my presentations and expectations, and how to ascertain and incorporate students’ preferred learning styles.

An additional teaching concern is represented by Adam, the student sitting at the front of the second row, arms folded across his chest, staring straight ahead. Around him, student voices buzz as other members of the class engage in small-group inquiry. Adam sits alone and waits. We both hear students challenging each other’s answers and debating the relative merits of different problem-solving strategies, but Adam remains motionless, even when I approach his desk. In response to my “How’s it going?,” there is neither a flicker of an eyelid nor a change in his position.

If waiting is the game, I can outwait any adolescent. I stand quietly, attentive to the industry around me, but I don’t move away from Adam. Finally, he pushes a sheet of binder paper toward me and points to a number written on it. When I neither move nor respond, Adam says, “The answer’s 13.” This is incorrect, but rather than tell him this, I ask, “How did you get that value?”

Without looking up, he tells me, “The work is so simple, any fool can see the answer.” That is a loaded reply, as I suspect he knows. To tell him he is incorrect is to risk suggesting that he is a fool. To give him the correct answer is to grant him tacit permission to remain disengaged.

As with many mathematics students who have preceded Adam, I am witnessing some of the methods employed by a student who may be bright, bored, and underachieving, in spite of his incorrect answer. The immediate problem is finding a way to spark Adam’s engagement. The larger problem is verifying Adam’s ability and, if my assumption is correct, developing effective teaching strategies to meet the needs of this potentially high-end learner in a heterogeneously grouped mathematics classroom.

## Heterogeneity

Schools face difficult decisions about the appropriate placement of students in mathematics classes (National Council of Teachers of Mathematics [NCTM], 2000). Although one of the educational goals for the United States was to be “first in the world in mathematics and science achievement by the year 2000” (Takahira, Gonzales, Frase, & Salganik, 1998, p. 17), the evidence from a variety of sources demonstrates that this goal has not been realized (Gonzales, Calsyn, Jocelyn, Mak, Kastberg, Arafeh, William, & Tsen, 2000; NCTM; Takahira et al.). Neither the practice of tracking nor that of heterogeneously grouping mathematics students has led to quantifiably higher standardized testing outcomes over the last decade.

Tracking, defined by Silver, Smith, and Nelson (1995) as placing middle and high school students in different mathematics classes based on ability, has led to unequal opportunities for students in the lower tracks to pursue higher level objectives. These authors relate that students left out of the higher track courses are denied access to high-quality, challenging mathematics. Compared to students who understand and can do mathematics, these lower tracked students have diminished opportunities and options for shaping their futures (NCTM, 2000).

The heterogeneously grouped classroom presents a different set of challenges. Here, mathematics teachers work with students who evidence a wide range of abilities and prior knowledge (Mills, Ablard, & Gustin, 1994; Van Tassel-Baska, 1991). The more variety exhibited by a group of students, the greater the potential challenge educators face in meeting their instructional needs. At what level should one teach in order to match curriculum with ability and to build from prior knowledge?

Clark (1997) suggested that the optimal environment for each student would be one where the level and pace of instruction is individually matched to the student. In reality, individual instruction is rarely possible in public school classrooms, where teachers usually work with large groups of students (Renzulli & Pu rcell, 1996). Teaching to the lower level of a class perpetuates the problem of low mathematics achievement, along with boredom and disengagement on the part of the middle and high-end learners. Teaching to the middle level causes the less-prepared students to struggle and fall farther behind, while the better prepared students, who remain unchallenged, lose their motivation to learn (Rimm & Lovance, 1992). Teaching to the high end also seems untenable, given the probable struggle and likely disengagement by less-prepared students.

Without changes in the level of classroom teaching, the outlook for promising mathematics students is bleak.

According to Rimm and Lovance (1992), “if we don’t provide a challenging environment, we are, in a *de facto* way, teaching our children to underachieve” (p. 10). Perhaps disengaged students like Adam are one result of the failure to teach at a level appropriate to high-end mathematics students, a failure has been documented. Shore and Delcourt (1996) found “that when gifted children were heterogeneously grouped within classes, they received less than 20% of the teacher’s attention and no curricular differentiation in 84% of their learning activities” (p. 142). They also reported that, “at best, only minor modifications to the regular curriculum were made for gifted students, even when there was a formal within-class gifted program in these schools” (p. 142).

## Mathematical Giftedness

The basis for any discussion regarding teaching at a level appropriate for mathematically gifted students begins with a general understanding of giftedness before moving to a specific understanding of mathematical giftedness, which is difficult because there is no universally accepted definition of general giftedness (Gagné, 1995; Morlock, 1996; Sernberg, 1993).

This fundamental lack of agreement extends to mathematics, where differing descriptors of high mathematical performance and ability are evident in the literature (Sowell, 1993). Sowell, Bergwall, Zeigler, and Cartwright (1990) documented a variety of literature-based adjectives to describe exceptional mathematics students. These descriptors include “promising,” “high-end learners,” “gifted and talented,” and “academically superior.” This multiplicity of descriptors within the specific domain of mathematics parallels the plurality of descriptors of giftedness in general.

Despite such different descriptions of mathematics students with high potential, the literature discussing these students (Sowell et al., 1990) agrees that mathematically gifted students are able to do mathematics typically accomplished by older students or engage in qualitatively different mathematical thinking than their classmates or chronological peers. This literature also frames a picture of mathematical talent that corresponds to an understanding of giftedness as a dynamic and emerging trait. The NCTM Task Force on Mathematically Promising Students (Sheffield, 1999) recognized that mathematically gifted students come in all sizes, ages, and levels of academic achievement and noted that they may not possess identical traits. Furthermore, the task force avoided defining mathematical promise as giftedness. Instead, they defined mathematically promising students as “those who have the potential to become the leaders and problem solvers of the future” (Sheffield, p. 9).

The difficulties of the task force's definition of "mathematical promise" lie in recognizing and nurturing potential. Are mathematically promising students those who accurately solve demanding problems, those who do mathematics typically accomplished by older students, those who demonstrate both these characteristics, or those who evidence some other combination of mathematical attributes? Rather than debate whether mathematically promising students are gifted, for the purposes of this discussion, mathematical giftedness is regarded as an emerging promise or high ability with mathematics relative to one's peers. In addition, this discussion accepts that high-ability mathematics students may not demonstrate their abilities consistently. Over time, however, they exhibit clusters of classroom behaviors that are markedly different from their classmates.

## Behaviors

Sowell, Zeigler, Bergwall, and Cartwright (1990) found that there are at least two types of mathematically gifted students. One type is the precocious student, able to do the mathematics typically accomplished by older students. The other type is the student who is able to solve demanding problems by employing qualitatively different thinking processes. Generally speaking, regardless of membership in either of these or other groups, highly able students acquire basic skills rapidly, reason quickly, and have the ability to form comprehensive generalizations more advanced than their agemates (Johnson, 1994).

While promising mathematics students will not evidence all traits, additional traits include longer attention spans, better memories, and greater persistence in wanting to find the solution to problems when compared to agemates (Garofalo, 1993). Some of these students may consistently create numerically inaccurate answers since they may spend relatively more time on the planning stages of problem solving and be less concerned about accuracy of calculations (Garofalo).

With no set of traits describing high-end mathematics students, it is evident that no single method of instruction necessarily addresses the needs of these students. Since Clark's (1997) suggestion of individualizing instruction is untenable and the range of needs can be great in the heterogeneously grouped mathematics classroom, differentiation presents an attractive answer to the dilemma of what a teacher can do. Returning to Adam's geometry class illustrates the power of this method.

## Differentiation

Adam was a student in one of the 17 heterogeneously grouped, non-honors geometry classes in a large suburban high

school. Twenty-two teachers formed the mathematics department, neither larger nor smaller than most mathematics departments in the more than two-dozen high schools in this district. As part of one of the nation's largest school districts, Adam's school was experiencing the pressures that accompany explosive population growth. Already racially diverse, it was suffering from severe overcrowding. Classes were "capped" at 32, based on the size of the rooms. In reality, class sizes often grew above this cap. New students were entering the district on an average of 300 per week. They had to be assigned to schools and receive schedules. In Adam's class, the number of students exceeded and receded several times from the stated cap of 32.

Clearly, the needs of such a class were dynamic. At times, I needed to present new information to the class using direct instruction. At other times, I could maximize student focus and mastery by creating small groups for investigations, for practice, or for compacting materials new students needed to master. During these small-group sessions, I was able to move around the class, listening to student discussions, providing scaffolding, asking open-ended questions, and assessing progress. Structured appropriately, these group tasks met many of the NCTM (2000) recommendations for better mathematics teaching.

However, group work, per se, does not represent differentiation, even when students are working on different problems (Hoeflinger, 1998). For true differentiation to occur, the teacher should preassess understandings central to a unit and then purposefully modify activities to eliminate repetition and drill for those who already demonstrate mastery. These modifications fall into three general categories: differentiated content, differentiated process, and differentiated product (Tomlinson, 1999). The key components of modifications to the mathematics curriculum should attend to four broad principles: The teacher should (1) provide content with greater depth and higher complexity, (2) nurture a discovery approach that encourages students to explore concepts, (3) focus on providing complex open-ended problems, and (4) create opportunities for interdisciplinary connections (Stepanek, 1999).

Adam's class presented a full range of student abilities and interests. Two of the students qualified for Honors Geometry, but declined to take that accelerated course. As the year passed, I discovered another three whom I believed exhibited high mathematics potential. Three students were mainstreamed learning-disabled students with IEPs. Several students had barely passed Algebra, a prerequisite in this district for Geometry, so they struggled with the mathematics behind many of the year's units.

The adopted text was *Geometry: An Integrated Approach* (Larson, Boswell, & Stiff, 1995), which followed a standard sequence of geometric topics. After an overview of the subject, the text provided a review of basic algebra and reason-

ing skills. Immediately thereafter, the focus shifted to geometry, beginning with the study of triangles. I used the study of triangles as a foundational unit that carried classically important mathematics and served as a vehicle for differentiation experiences.

Over the course of the unit, I was able to provide three different types of differentiation. The first was extension, the second was open-ended investigation, and the third was self-selection of problems. Each differentiation opportunity provided an opportunity for content differentiation, process differentiation, and product differentiation.

### Modification 1: Extension and Application

Adam's class began their study of triangles in October. The two major ideas that anchored the fall curriculum were congruency and the Pythagorean theorem. The class began the study of congruency by learning about the different kinds of triangles. This required that they understand the descriptive attributes of triangles and correctly apply them. Adam and four other students grasped the descriptors of triangles, could apply them accurately, and needed more or different work in order to increase their mathematical understandings.

The initial modification presented to these five students was an extension of the task of defining and applying attributes to a group of shapes. Their task was to create at least two systems to describe and sort quadrilaterals. The students were to test their proposed systems, modify them as needed, and present their findings in two forms to the full class. They could work as one large group or as two small groups. No one could work alone.

My expectation was that these students would discuss and then organize all the quadrilateral shapes into two groups that paralleled the categorization of triangles by angle or length of side. Instead, they found the flaw in the assignment within moments of starting their considerations. Although they did not have the mathematical vocabulary, they discovered convex and concave quadrilaterals. After a lengthy debate among themselves and one short conference with me, they decided to include both types of quadrilaterals in their discussions because they wanted to be exhaustive in their considerations.

These students were ready to present their findings to the class before the rest of the students had reached a good place to stop their work. To allow these five to continue creating meaningful learning, I asked each of them to pose at least three questions about their new understandings. They were instructed to pool their questions, arrange them in a hierarchy from most to least important, write them on chart paper, post the papers to form an "inquiry wall," and begin researching the questions. Hypotheses or answers to these questions were

added to the chart paper over the course of the next several months.

This wall became integral to the learning experience of the whole class. At different times during the fall semester, every member of the class contributed to the growing body of displayed information by adding questions, suggesting hypotheses, or providing answers. The wall became the foundation of self-selected, but focused, inquiry for the whole class during the fall semester.

### Modification 2: Investigating an Open-Ended Question

As the class began to study the nature of congruency, Adam and two others from the first modification group immediately demonstrated an intuitive understanding of the pieces necessary to prove triangles congruent. Preassessment showed that another student not from the original five also understood. These four became a group that investigated the following open-ended question: What is the minimum information necessary to prove two triangles are congruent?

In pairs, the students proposed different ways to prove congruency. They tested the need to prove each of the ways they had identified. Then, the two pairs debated each other and sought counterexamples. They discovered that they could prove right triangle congruency with less information than they needed for all other triangles, and they discovered the ambiguous case that is usually held over until trigonometry. They demonstrated their new understandings by conferencing with me.

I posed one question that led to a presentation to the whole class: I asked the four if there was any idea they thought was interesting enough to share with their classmates. They thought the ambiguous case was "fun" and would clarify the common misconception that side-angle-angle proved congruency. They made a 15-minute presentation to the full class after the congruency unit test.

### Modification 3: Self-Selection of Problems

The study of the Pythagorean theorem provided another opportunity for differentiation, this time for eight students. These students had easily mastered the application of the Pythagorean theorem, as well as the adaptations available for proving right triangle congruency. Not surprisingly, the strengths and interests of this larger group varied more than the strengths of the smaller groups.

To address this wider range of interests and needs, I allowed the students to self-select from a menu of opportunities. Their choices included exploring the history of the Pythagorean theorem; exploring at least three ways of proving the Pythagorean theorem; exploring different kinds of proofs,

plus the difference between proof and demonstration; and exploring the nature of square roots, including how to visualize them. Those students who had selected the same topic worked together. Otherwise, a student worked alone. All were instructed to create some kind of poster or large visual, plus a short written explanation of their findings. They later presented these during a poster session held in conjunction with a series of research project presentations by the remaining members of the class.

### Discussion of Modifications

These three modifications were linked to one long unit on triangles that I taught in the fall. Throughout the year, every unit presented additional points at which the same students demonstrated their readiness to move on to different material. At no time did I attempt to offer differentiation on a daily or task-by-task basis. Instead, I worked with the major concepts and skills that anchored each unit. Those who demonstrated mastery of these concepts and skills were invited to move into a differentiated option that was linked to the material being studied by the rest of the class.

The differentiation strategies employed in Adam's geometry class were not limited to a particular group of students (Stepanek, 1999). All students were eligible to participate in each modification based on demonstrated readiness. I prepared the modifications for points in the unit where I believed high-end students could become bored while their classmates worked more slowly. At these points in the units, all students were invited to demonstrate their understanding of the ideas fundamental to the concepts being developed and to show mastery of the skills necessary to perform the required calculations.

The class understood, based on the opportunity afforded each class member to demonstrate understanding and mastery, that I was not preselecting favored students for inclusion in some special group. The class also understood that the differentiation opportunities were not a pause from learning important mathematics. These differentiated opportunities became known with humor and a nod to Robert Frost as "The Road Not Taken—By Most." Five students were ready for the first differentiated opportunity. Three of the first five plus two others participated in the second differentiated opportunity. The original five, the additional two from the second opportunity, and one other student demonstrated readiness for the third opportunity.

The advantages of the differentiated opportunities seemed to be understood by all. The high-end learners did not have to wait for their classmates before moving forward. They were able to work with more abstract material, such as the ambiguous

case, and at a pace more aligned with their understandings. Yet, the varied presentations, posters, questions, hypotheses, evidence, and answers allowed all students to have access to the ideas that were considered by the small groups and individual investigators. Meanwhile, the other students progressed with their own learning, secure in the understanding that I was attentive to their needs. Each modification reflected Stepanek's (1999) ideas that differentiation should provide content with greater depth and higher complexity; nurture a discovery approach that encourages students to explore concepts; provide complex, open-ended problems; and create opportunities for interdisciplinary connections.

Likewise, the modifications attended to content, process, and product differentiation (Tomlinson, 1999). Content differentiation occurred with each modification. When the students posed, sought, and answered their own questions during the first modification, they were creating their own content extensions. Later, as the next group discovered, investigated, and presented the ambiguous case to the class, they created their own content differentiation. They found the ambiguous case as a result of their open-ended explorations to establish the minimum information needed to prove triangle congruency.

The third modification provided the most variety and personal selection for the students. The four choices provided opportunities for abstraction, for the study of history and philosophic differences found in standards of evidence, and for concrete and tactile creations linking complex thought with real outcomes. The students chose from among the following: connecting with the history of a major topic in mathematics, considering different methods for proving the Pythagorean Theorem, discovering the fundamental properties of proofs and contrasting these with the properties of demonstrations, and examining and seeking physical representations of radicals using geoboards.

Process differentiation occurred with all three modifications. Debate, conferencing, creating oral presentations and visual support materials, researching history, investigating the components necessary for proving theorems and conjectures, experimenting with ways to demonstrate irrational numbers (many radicals)—each of these provided the students with rich, open-ended options from which they might create their own learning.

Product differentiation was also evident in each of the modifications. I required evidence of the students' new understandings for each modification. Sometimes this evidence was oral, sometimes visual, and sometimes written. Each time, the whole class enjoyed hearing about the work these students had done, although the class was not required to develop the same level of understanding about the various topics as the investigators were.

## Conclusions

The original question asked, "What's a teacher to do?" The heterogeneously grouped mathematics classroom presents a wide range of student interests and abilities. Attempting to teach to any single level in the class does not meet the needs of all students. Moving too slowly or with low-level material will not lead to any improvement in the aggregate mathematical abilities of students as measured by standardized tests or international research studies. Teaching to the high-end learners risks engaging in material too abstract for the majority of the students or in pacing instruction too quickly for them. Regardless of the level at which we teach the whole class, we risk ignoring the needs of some portion of it. Differentiation provides a solution to the dilemma, a solution that can provide appropriate challenges at appropriate levels for all learners.

Differentiation is not an exclusionary tactic. As practiced in Adam's class, all students were eligible to participate in each modification. To meet the needs of all my students, I needed to assess their readiness to move on. The modifications provided differentiation for those who were ready to move ahead in their learning. The modifications did not allow students to skip important conceptual understandings or skill acquisition. Instead, they underscored the importance of students continuing to learn important mathematics. The students who proved ready to move into a modification were responsible for respectful learning and were required to demonstrate their new understandings.

Taken together, the three modifications to the unit on triangles and the Pythagorean theorem attended to the four principles described by Stepanek (1999). The students who engaged in the modifications worked with content that had greater depth and complexity than the work assigned to the rest of the class. The students who engaged in the modifications used a discovery approach that encouraged exploration. The topics provided complex, open-ended problems. The third modification created specific opportunities for interdisciplinary connections.

What happened to Adam, the disengaged student we met at the start of this discussion? Did differentiation make a difference to his learning? I wish I could report that he became focused and excited about geometry in particular, mathematics in general, and learning for all time. He did not.

Although he qualified for all three of the modifications, he spent the entire year actively trying to remain disengaged. He argued that I was unfair to him when I expected him to learn materials that were different from those most of the class was studying. We conferenced about this point many times during the year, and each time Adam admitted that he already knew the material contained in the unit. Nevertheless, he would debate the rationale for being required to engage in new

learning each time he demonstrated that he was ready to move forward into a modification.

Adam grudgingly participated in the first two modifications, and his work was excellent. He refused to select one of the options offered for the third differentiation. I would not assign one. His counselor and parents became involved at that point. Thereafter, he worked independently and well, creating an excellent product.

Based on the high level of his work and his yearlong resistance to the many opportunities to participate in differentiated learning, I concluded that the options Adam experienced with me in his ninth grade were too little, too late. Although he was capable of high-quality mathematics, he was content to remain disengaged and unchallenged. I will always wonder what might have occurred if Adam had encountered differentiated work much earlier in school.

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